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STRESS ANALYSIS AND FAILURE OF AN INTERNALLY PRESSURIZED COMPOSITE-JACKETED STEEL CYLINDER

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SUMMARY

This paper presents a nonlinear stress analysis of a thick-walled compound tube subjected to internal pressure. The compound tube is constructed of a steel liner and a graphite-bismaleimide outer shell. Analytical expressions for the stresses, strains, and displacements are derived for all loading ranges up to failure. Numerical results for the stresses and the maximum value that the compound tube can contain without failure are presented.

INTRODUCTION

Weight reduction is a requirement for a majority of weapon systems being developed by the Army. The Army would like to design a longer cannon and maintain the inertia characteristics of the shorter cannon. This accomplishment would allow current cannon mounts to be used. The longer cannon is expected to achieve higher muzzle velocity and greater accuracy than the standard cannon. The design under consideration is to replace a portion of the steel wall thickness with a lighter material. The inner portion, the steel liner, maintains the tube projectile interface and shields the composite from the extremely hot gases. The outer portion, the composite jacket, is made of a fiber-reinforced organic composite (graphite fiber and a bismaleimide matrix). A linear stress analysis for this problem under internal pressure in the elastic range was reported in a recent paper by M.D. Witherell and M.A. Scavullo (ref. 1).

This paper presents a nonlinear stress and failure analysis of the compound tube problem. The loading ranges include elastic, elastic-plastic, and fully-plastic up to failure. Analytical expressions for the stresses, strains, and displacements are derived for all cases. Numerical results for the radial and hoop stresses in the nonlinear loading ranges are presented. The maximum value of internal pressure that the compound tube can contain without failure is predicted.

PROBLEM AND ELASTIC ANALYSIS

Figure 1 shows a schematic of the compound tube problem. The compound tube consists of an inner steel "liner" and an outer composite "jacket." The steel liner of inside radius a and outer radius

b is wrapped in the circumferential direction with a graphite-bismaleimide organic composite of outside radius c. The elastic material constants for the composite and the steel are given in Table I.

TABLE I. ELASTIC CONSTANTS OF COMPOSITE JACKET AND STEEL LINER

Elastic Constants for IM6/Bismaleimide, 55% F.V.R.		
E_r = 1.126 Mpsi E_θ = 23.31 Mpsi E_z = 1.126 Mpsi	$\nu_{r\theta}$ = 0.01524 $\nu_{\theta z}$ = 0.3155 ν_{zr} = 0.3991	$\nu_{\theta r}$ = 0.3155 $\nu_{z\theta}$ = 0.01524 ν_{rz} = 0.3911
Elastic Constants for Steel		
E = 30.0 Mpsi	ν = 0.3	

When the composite tube is subjected to internal pressure p in the elastic range, the general solutions in the plane-strain condition for the isotropic liner ($a \leqslant r \leqslant b$) are

$$\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix} = \{\mp(p-q)(\frac{b}{r})^2 + p - q\ b^2/a^2\}/(\frac{b^2}{a^2} - 1) \qquad \begin{matrix}(1) \\ (2)\end{matrix}$$

$$u/r = E^{-1}(1+\nu)[(p-q)(b/r)^2 + (1-2\nu)(p-q\ b^2/a^2)]/(\frac{b^2}{a^2} - 1) \qquad (3)$$

and for the orthotropic jacket ($b \leqslant r \leqslant c$),

$$\sigma_r = q[-\ (\frac{c}{b})^{k-1}(\frac{c}{r})^{k+1} + (\frac{r}{b})^{k-1}]/[(c/b)^{2k} - 1] \qquad (4)$$

$$\sigma_\theta = kq[(c/b)^{k-1}(\frac{c}{r})^{k+1} + (\frac{r}{b})^{k-1}]/[(c/b)^{2k} - 1] \qquad (5)$$

$$u/r = \epsilon_\theta = \alpha_{12}\sigma_r + \alpha_{22}\sigma_\theta \qquad (6)$$

where q is the pressure at the interface, $k = (\alpha_{11}/\alpha_{22})^{½}$,

$$\alpha_{11} = (1-\nu_{rz}\nu_{zr})/E_r$$

$$\alpha_{12} = -\ (\nu_{\theta r}+\nu_{\theta z}\nu_{zr})/E_\theta$$

$$\alpha_{22} = (1-\nu_{\theta z}\nu_{z\theta})/E_\theta \qquad (7)$$

By requiring the displacement to be continuous at the interface, the interface pressure q can be expressed as a linear function of internal pressure p,

$$\frac{2p}{q} = (\frac{b^2}{a^2} - 1)[Ak\ \frac{(c/b)^{2k} + 1}{(c/b)^{2k} - 1} + B] + \frac{b^2}{a^2} + 1 \qquad (8)$$

where

$$A = E\alpha_{22}/(1-\nu^2),\ B = -E\alpha_{12}/(1-\nu^2) - \nu/(1-\nu) \tag{9}$$

Now all the stresses, strains, and displacements in the tube ($a \leqslant r \leqslant c$) can be determined as functions of p. In particular, the expressions for the displacements at the bore (u_a), interface (u_b), and outside surface (u_c) are

$$\left(\frac{b^2}{a^2} - 1\right)\frac{E}{p}\frac{u_a}{a} = (1+\nu)\frac{b^2}{a^2} + (1-\nu-2\nu^2) - \frac{4(1-\nu^2)(b^2/a^2)}{\left(\frac{b^2}{a^2} - 1\right)\left[AK\frac{(c/b)^{2k}+1}{(c/b)^{2k}-1}+B\right] + \frac{b^2}{a^2} + 1} \tag{10}$$

$$\frac{u_b}{b} = q\left[k\alpha_{22}\frac{(c/b)^{2k}+1}{(c/b)^{2k}-1} - \alpha_{12}\right] \tag{11}$$

$$\frac{u_c}{c} = \frac{2qk\alpha_{22}(c/b)^{k-1}}{(c/b)^{2k}-1} \tag{12}$$

ELASTIC-PLASTIC ANALYSIS

When the internal pressure p is large enough, part of the steel liner will become plastic. Using Tresca's yield criterion, the associated flow rule, and assuming linear strain-hardening, the elastic-plastic solution based on Bland can be used (ref. 2 or ref. 3). Let ρ be the elastic-plastic interface.

The solution can be written in the elastic portion ($\rho \leqslant r \leqslant b$) as

$$\frac{E}{\sigma_o}\frac{u}{r} = \frac{1+\nu}{2}\frac{\rho^2}{r^2} + (1-\nu-2\nu^2)\left[\frac{1}{2}\frac{\rho^2}{b^2} - \frac{q}{\sigma_o}\right] \tag{13}$$

$$\left.\begin{matrix}\sigma_r/\sigma_o \\ \sigma_\theta/\sigma_o\end{matrix}\right\} = \frac{1}{2}\left(\mp\frac{\rho^2}{r^2} + \frac{\rho^2}{b^2}\right) - \frac{q}{\sigma_o} \qquad (14),\ (15)$$

$$\sigma_z/\sigma_o = \nu\ \rho^2/b^2 - 2\nu\ q/\sigma_o \tag{16}$$

and in the plastic portion ($a \leqslant r \leqslant \rho$)

$$\frac{E}{\sigma_o}\frac{u}{r} = (1-\nu-2\nu^2)\frac{\sigma_r}{\sigma_o} + (1-\nu^2)\frac{\rho^2}{r^2} \tag{17}$$

$$\left.\begin{matrix}\sigma_r/\sigma_o \\ \sigma_\theta/\sigma_o\end{matrix}\right\} = \mp\frac{1}{2}\left(1-\eta\beta+\eta\beta\frac{\rho^2}{r^2}\right) + \frac{1}{2}\frac{\rho^2}{b^2} - (1-\eta\beta)\ln\frac{\rho}{r} - \frac{q}{\sigma_o} \qquad (18),\ (19)$$

$$\sigma_z/\sigma_o = \nu\ \rho^2/b^2 - 2\nu(1-\eta\beta)\ln\frac{\rho}{r} - 2\nu\ q/\sigma_o \tag{20}$$

$$\bar{\epsilon}^p = \beta(\rho^2/r^2-1)\quad,\quad \eta\beta = \frac{m}{m + \frac{3}{4}\frac{(1-m)}{(1-\nu)^2}} \tag{21}$$

$$\eta = \frac{2}{\sqrt{3}}\frac{E}{\sigma_o}\frac{m}{1-m}\quad,\quad m = \frac{E_t}{E}\quad,\quad \sigma = \sigma_o(1+\eta\epsilon^p) \tag{22}$$

where σ_o is the initial tensile yield stress, and E_t is the tangent modulus in the plastic range of the stress-strain curve.

Using Eqs. (11) and (13) and the requirement of displacement continuity at the interface, i.e., u_{b-} (liner) = u_{b+} (jacket), we obtain the expression for the interface pressure q as

$$\frac{q}{\sigma_o} = \frac{(1-\nu^2)\rho^2/b^2}{(1+\nu)(1-2\nu) + E[\alpha_{22}k\,\frac{(c/b)^{2k}+1}{(c/b)^{2k}-1} - \alpha_{12}]} \tag{23}$$

Given any value of ρ in $a \leqslant \rho \leqslant b$, we can now determine q, u, and all the stresses and strains in the tube. In particular, the expressions for internal pressure and for displacements at the bore and the interface are

$$\frac{p}{\sigma_o} = \frac{q}{\sigma_o} + \frac{1}{2}\left(1 - \frac{\rho^2}{b^2}\right) + (1-\eta\beta)\ln\frac{\rho}{a} + \frac{1}{2}\eta\beta\left(\frac{\rho^2}{a^2} - 1\right) \tag{24}$$

$$\frac{E}{\sigma_o}\frac{u_a}{a} = -(1-\nu-2\nu^2)\frac{p}{\sigma_o} + (1-\nu^2)\rho^2/a^2 \tag{25}$$

$$\frac{E}{\sigma_o}\frac{u_b}{b} = (1-\nu^2)\frac{\rho^2}{b^2} - (1-\nu-2\nu^2)\frac{q}{\sigma_o} \tag{26}$$

By letting ρ = a and b, we can determine the lower limits p^*, q^*, $u_a{}^*$, $u_b{}^*$, $u_c{}^*$, and the upper limits p^{**}, q^{**}, $u_a{}^{**}$, $u_b{}^{**}$, and $u_c{}^{**}$, respectively.

FULLY-PLASTIC ANALYSIS

When the internal pressure p is further increased, i.e., $p > p^{**}$, the steel liner will become fully-plastic. The composite jacket remains elastic as long as the failure pressure is not reached. Using Tresca's yield criterion, the associated flow rule, and assuming linear strain-hardening, a fully-plastic solution can be obtained (ref. 4). The result is presented here for completeness. The explicit expressions for the displacement, strains, and stresses in the plane-strain case, subject to $\sigma_\theta \geqslant \sigma_z \geqslant \sigma_r$, are

$$ru = E^{-1}(1-2\nu)(1+\nu)r^2\sigma_r + \phi\, b^2 \tag{27}$$

$$\sigma_r = -p + \sigma_o(1-\eta\beta)\ln(\frac{r}{a}) + \frac{1}{2}\frac{\eta\beta}{(1-\nu^2)}[\frac{b^2}{a^2} - \frac{b^2}{r^2}]E\phi \tag{28}$$

$$\sigma_\theta = \sigma_r + \sigma_o(1+\eta\bar{\epsilon}^p) \tag{29}$$

where σ_o, η, $\bar{\epsilon}^p$ are the initial yield stress, hardening parameter, and equivalent plastic strain, respectively, and

$$\bar{\epsilon}^p = \frac{2}{\sqrt{3}}[\phi\, b^2/r^2 - (1-\nu^2)\sigma_o/E]/[1 + \frac{2}{\sqrt{3}}(1-\nu^2)\eta\sigma_o/E] \tag{30}$$

$$\phi = [E\alpha_{22}k\frac{(c/b)^{2k}+1}{(c/b)^{2k}-1} - E\alpha_{12} + (1-2\nu)(1+\nu)]q/E \tag{31}$$

$$p = \sigma_o(1-\eta\beta)\ln\frac{b}{a} + q\{1 + \frac{1}{2}\eta\beta(\frac{b^2}{a^2} - 1)[Ak\frac{(c/b)^{2k}+1}{(c/b)^{2k}-1} + B + 1]\} \tag{32}$$

It is interesting to point out that p is a linear function of q. Similarly, when evaluating u at the bore from Eq. (27), we obtain

$$u_a/a = -(1-2\nu)(1+\nu)P/E + \phi\, b^2/a^2 \tag{33}$$

which can also be expressed as a linear function of q with the aid of Eqs. (31) and (32). Since the relation between q and u_b is linear from Eq. (11), p and u_a, given by Eqs. (32) and (33), respectively, can be expressed as linear functions of u_b.

FAILURE ANALYSIS

Since the steel liner is ductile and failure precedes by plastic flows, a nonlinear stress analysis beyond the elastic limit is required. The liner is considered as failure when the maximum stress or maximum strain reaches the ultimate limit (σ_u or ϵ_u). The steel is assumed to be elastic-plastic, linear strain-hardening with σ_o = 120 Ksi, E_t = 120 Ksi, and σ_u (ultimate strength) = 140 Ksi. The composite jacket is elastically-orthotropic, and brittle failure is considered with the maximum strain criterion (ref. 5). The maximum strain from each simple test is either measured or computed from the measured strength divided by the elastic modulus. For the composite jacket used here, the maximum strain criterion is

$$-\epsilon_x'^* \leq \epsilon_\theta \leq \epsilon_x^* \quad \text{and} \quad -\epsilon_y'^* \leq \epsilon_r \leq \epsilon_y^* \tag{34}$$

where

$$\epsilon_x^* = X/E_\theta \quad , \quad \epsilon_x'^* = X'/E_\theta \quad , \quad \epsilon_y^* = Y/E_r \quad , \quad \epsilon_y'^* = Y'/E_r \tag{35}$$

and X, X', Y, Y' = 262, 225, 8.7, 21.8 Ksi, respectively.

DISCUSSION OF RESULTS

Given any value of internal pressure, we can obtain numerical results for the stresses and strains in the radial and tangential directions and also for the displacement at any radial position in a compound tube. The actual specimens were constructed (ref. 1) using steel liners with two thicknesses and the appropriate thickness of the composite circumferentially wound on the liner. The geometric dimensions (a,b,c) for the three composite tubes are (0.9, 1.0, 1.189), (0.9, 1.07, 1,189), and (0.9, 1.07, 1.391) inch. The pressure at the interface between the liner and jacket has been obtained as a function of internal pressure and the results for three cases are shown in Figure 2. In this figure we also show the limits of internal pressure in the elastic-plastic range, i.e., (p^*, p^{**}) = (20.48, 23.93), (23.06, 28.75), and (27.47, 34.98) Ksi, respectively. The results of the hoop strains at the bore, interface between the liner and jacket, and outside surface for case 2 with (a,b,c) = (0.9, 1.07, 1.189) inch are shown in Figure 3 as functions of internal pressure. The complete (including elastic, elastic-plastic, and fully-plastic) ranges of loadings up to failure have been considered. The maximum value of internal pressure that this compound tube can contain without failure is p_f = 48.483 Ksi, and the corresponding hoop strain is 1.12 percent. The results of hoop stresses at the bore ($\sigma_{\theta/a}$) and at the interface ($\sigma_{\theta/b-}$ and $\sigma_{\theta/b+}$) are shown in Figure 4 as functions of internal pressure. It should be noted that the hoop stresses at the interface are discontinuous with b- and b+ representing the location in the liner and jacket, respectively. Figure 4 shows very clearly that the results change drastically when yielding occurs. The relation changes from linear to nonlinear when yielding sets in and a more significant change occurs when the fully-plastic state is reached. The distribution of hoop stresses in the liner and jacket can be obtained at any given value of internal pressure. In Figure 5 we present the stress distributions for five values of internal pressure, i.e., p = 23.065, 26.638, 28.751, 36.617, and 48.483 Ksi. The first three values correspond to initial yielding, 50 percent yielding, and 100 percent yielding. The percent yielding in the elastic-plastic range is defined by $(\rho-a)/(b-a) \times 100$ percent. After the fully-plastic state is reached, the stress distribution changes drastically as shown in the figure for the last three values of internal pressure. The hoop stresses in the liner decrease slightly, but those in the jacket increase elastically as internal pressure is increased.

CONCLUSION

The stresses, strains, and displacements in the liner and jacket can be obtained analytically for all loading ranges up to failure. The plastic deformation in the liner has a significant effect on the overall performance of the composite structure.

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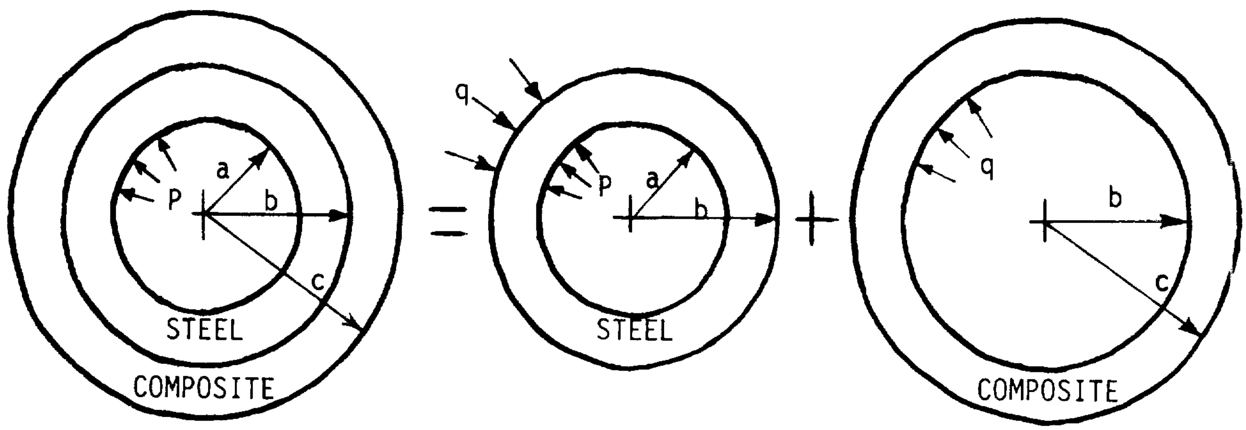

Figure 1 Schematic of a compound tube problem

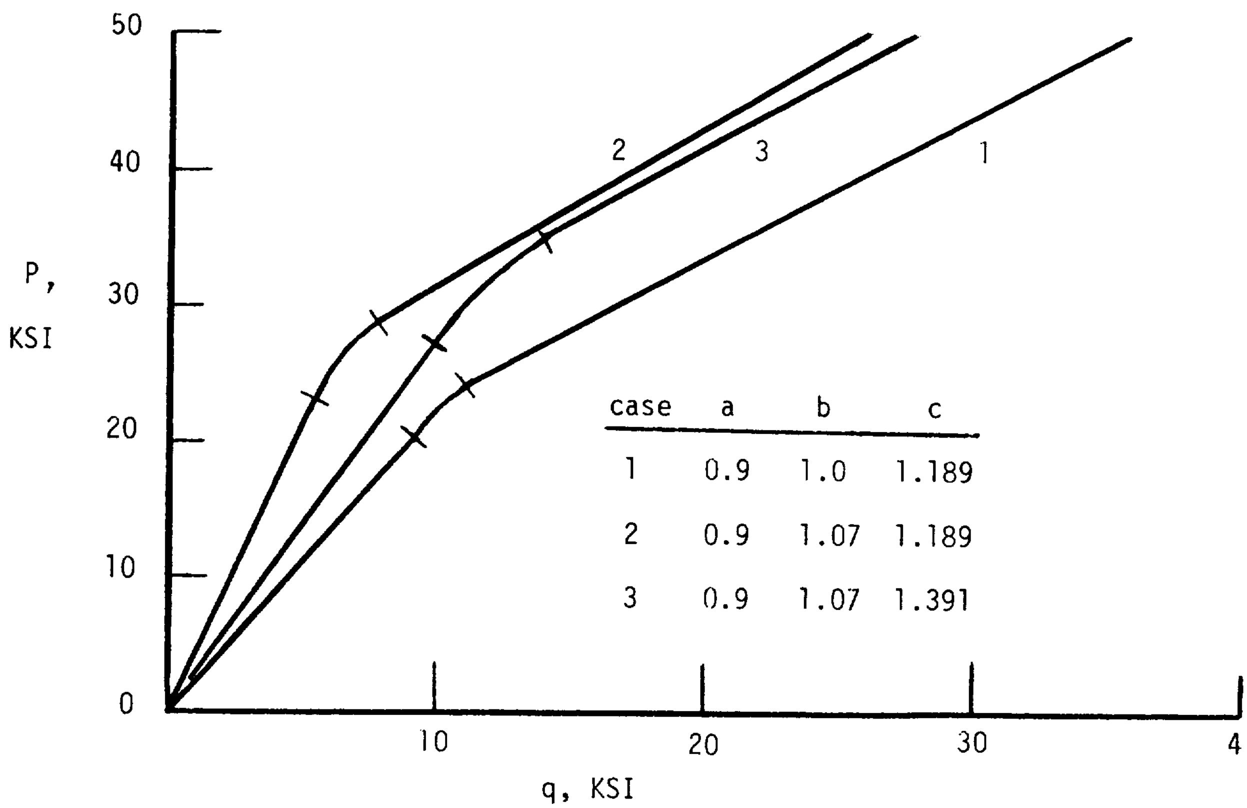

Figure 2 Interface pressure as a function of internal pressure

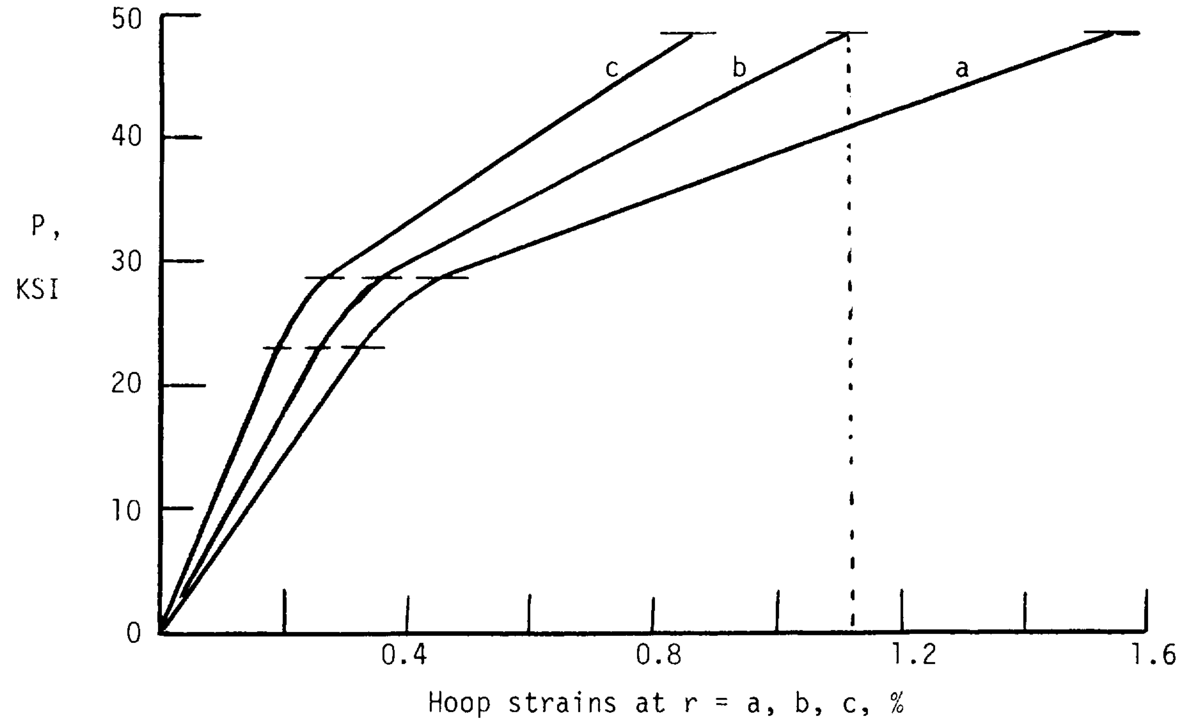

Figure 3 Hoop strains at the bore,interface and outside surface as functions of internal pressure.

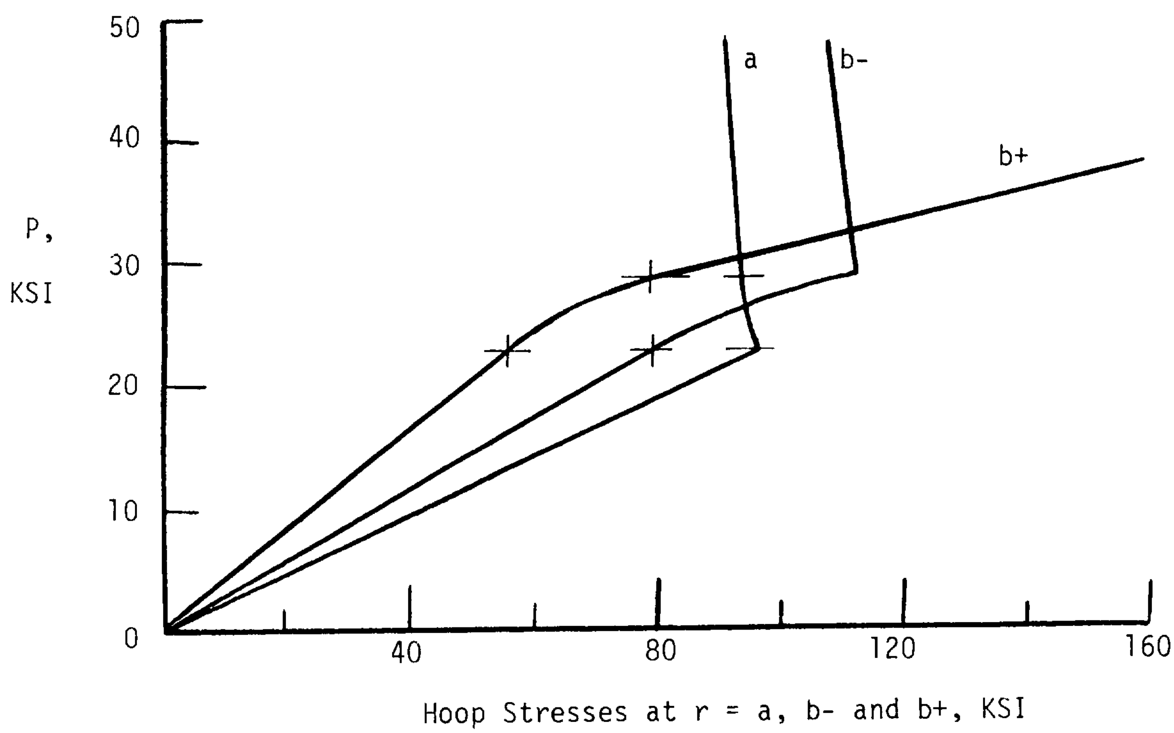

Figure 4 Hoop stresses at the bore and interface as functions of internal pressure

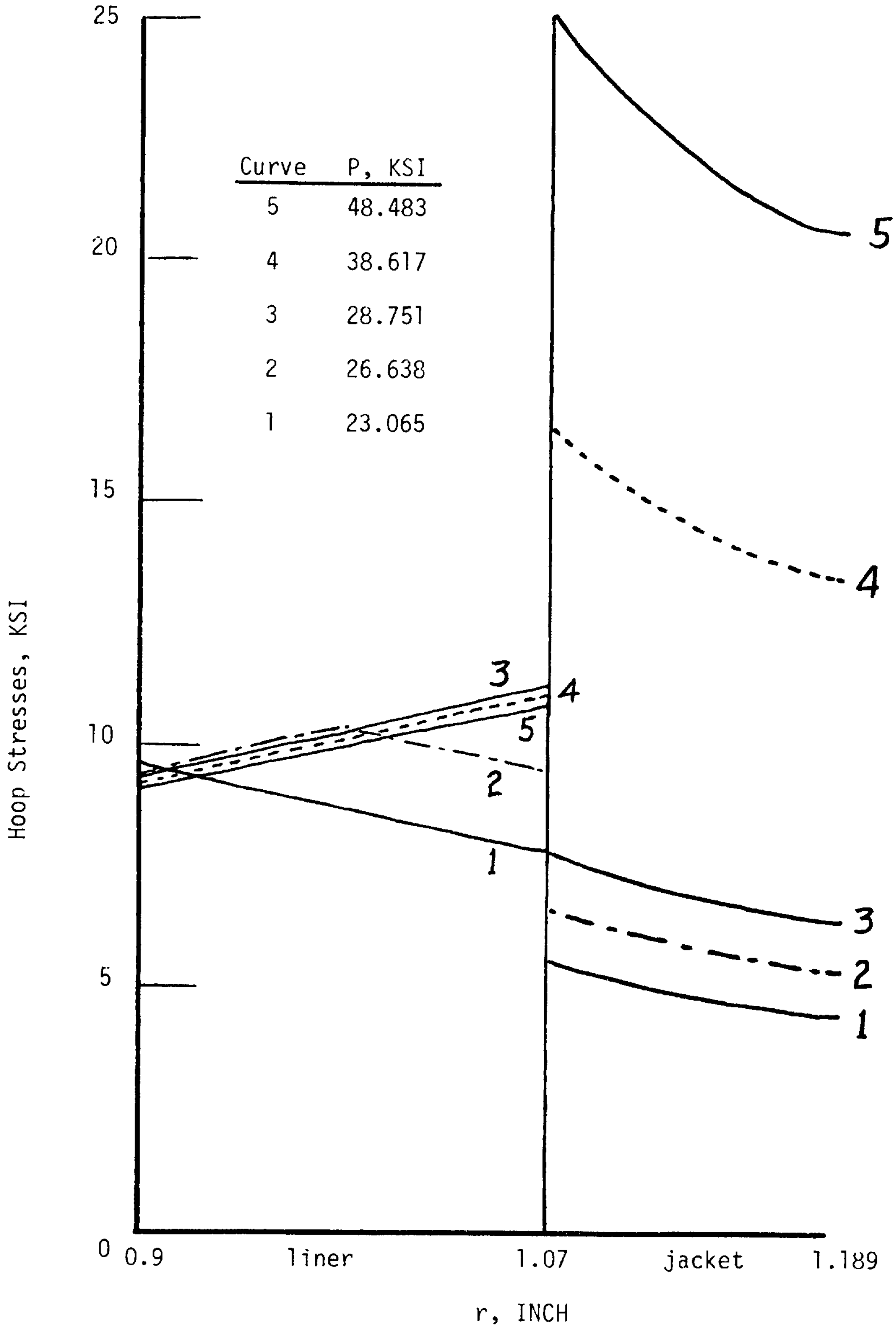

Figure 5 Distribution of hoop stresses in the liner and jacket